Optimal Transport Reconstruction of the Cosmic Web

Ravi K Sheth (UPenn)
with
Farnik Nikakhtar (Yale), Roya Mohayaee (IAP), Bruno Levy (INRIA), Nikhil Padmanabhan (Yale) and Sebastian von Hausegger (Oxford)

Motivation (Cosmology)

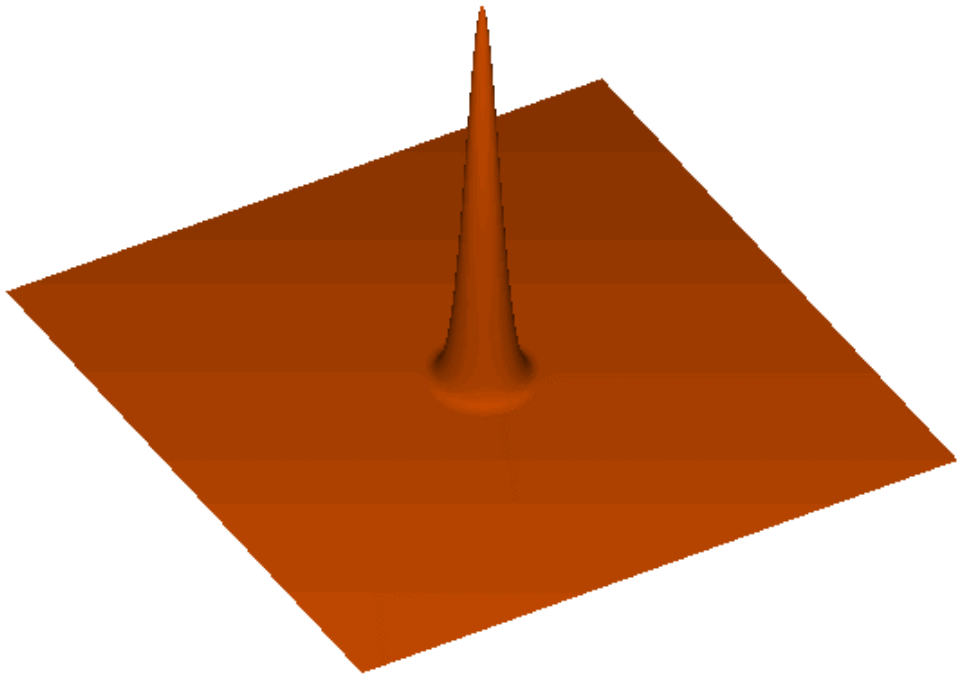
Reconstruction (OT)

Complications (Real-world)

CMB from interaction between photons and baryons when Universe was 3,000 degrees (about 380,000 years old)

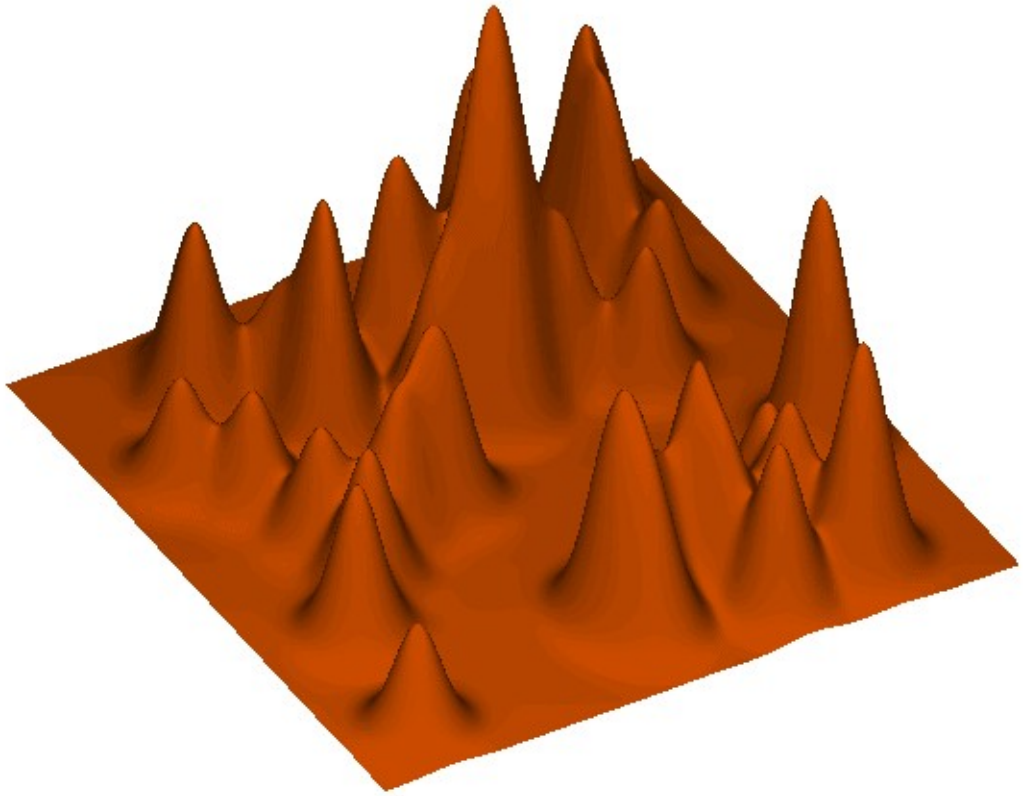
- Do galaxies which formed much later carry a memory of this epoch of last scattering?

Photons ‘drag’ baryons for ~400,000 years (time set by $\Omega_m h^2$) at speed ~ $c/[3(1 + 3\rho_b/4\rho_\gamma)]^{½}$ (set by $\Omega_b h^2$) … 300,000 light years ~ 100,000 pc ~ 100 kpc



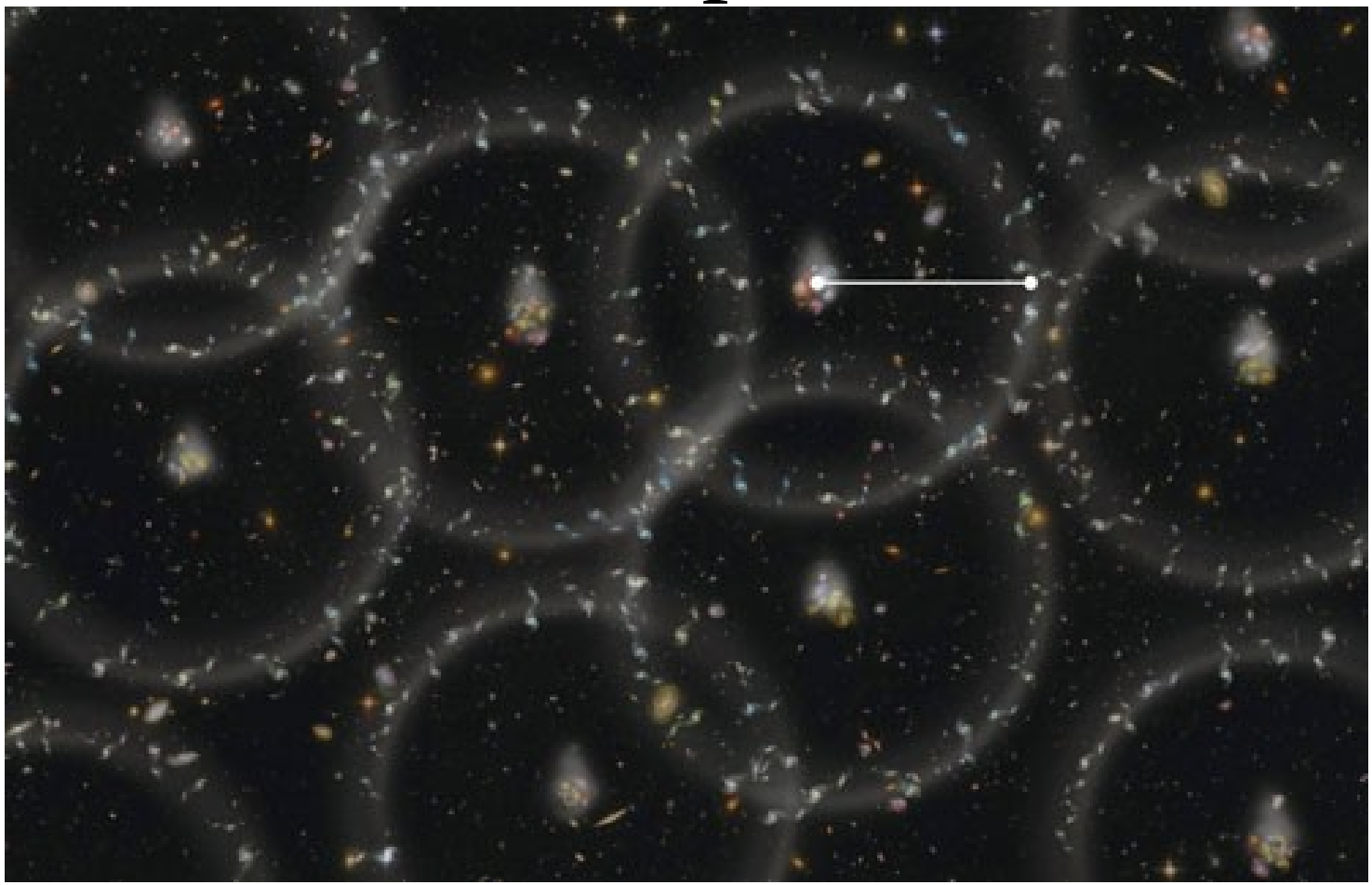
Expansion of Universe since then stretches this to (3000/2.725) ×100 kpc ~ 100 Mpc

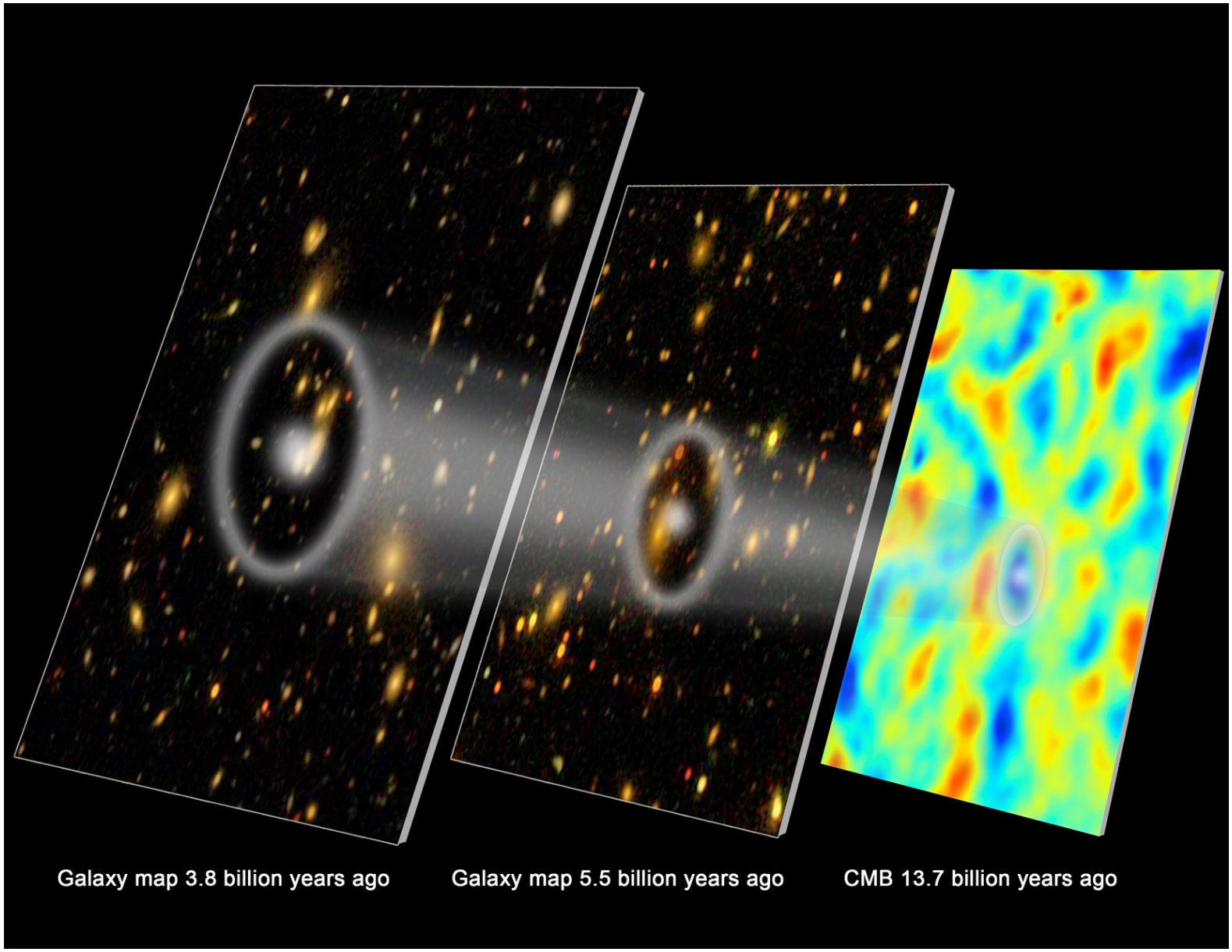
Expect to see a feature in the Baryon distribution on scales of 100 Mpc today



But this feature is like a standard rod:
We see it in the CMB itself at z~1000
Should see it in the galaxy distribution at other z

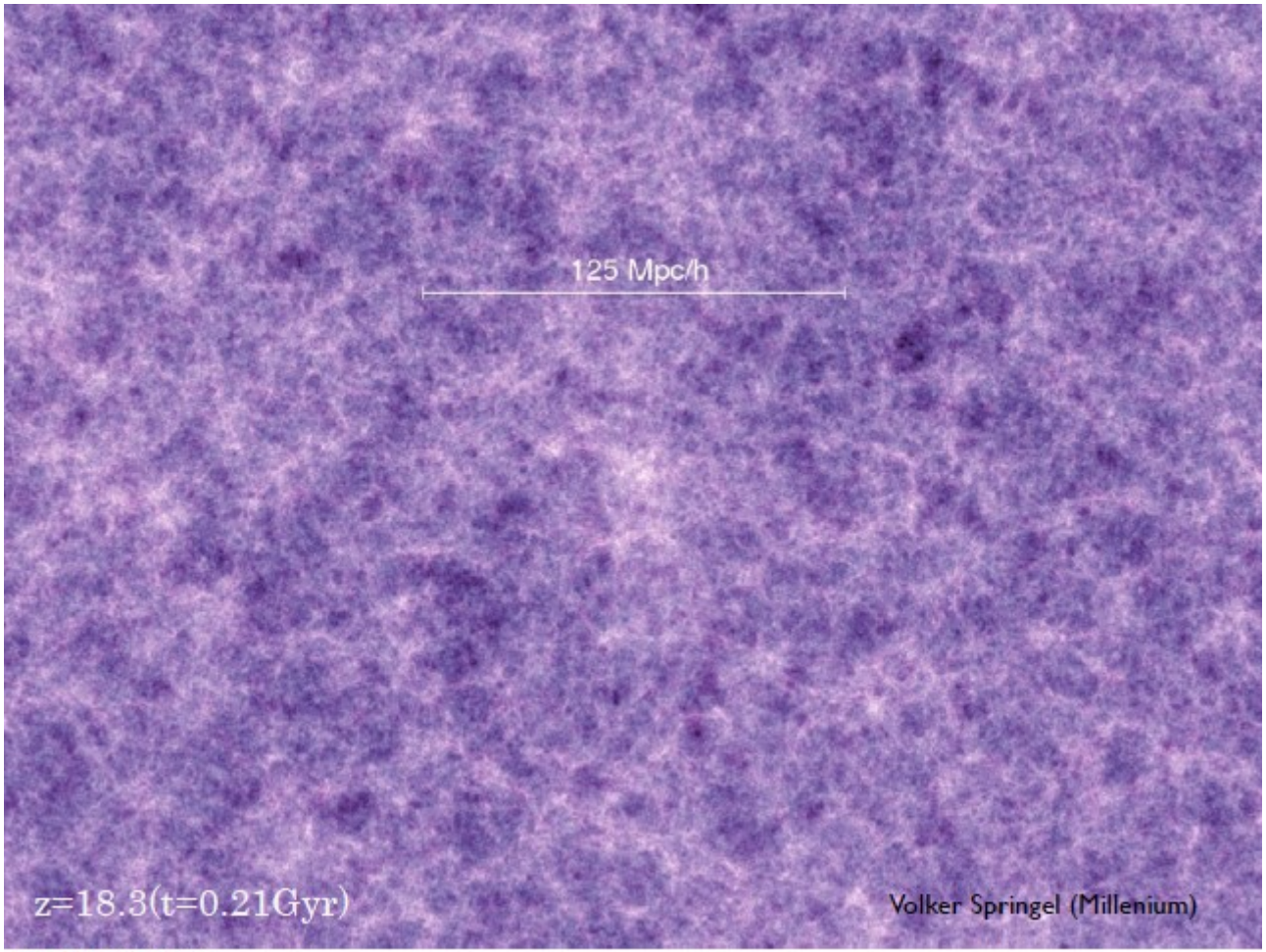
Cartoon of expected effect

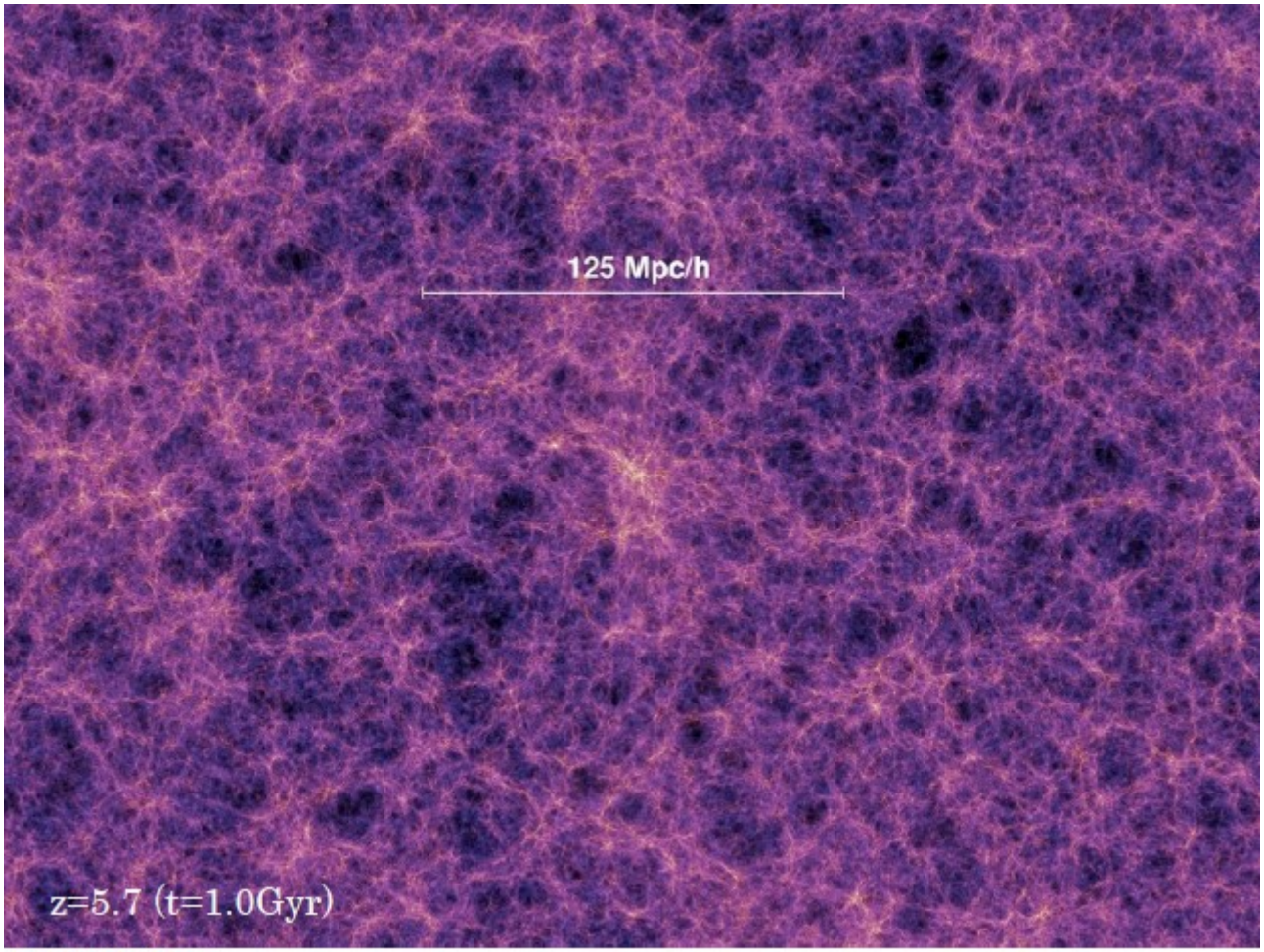


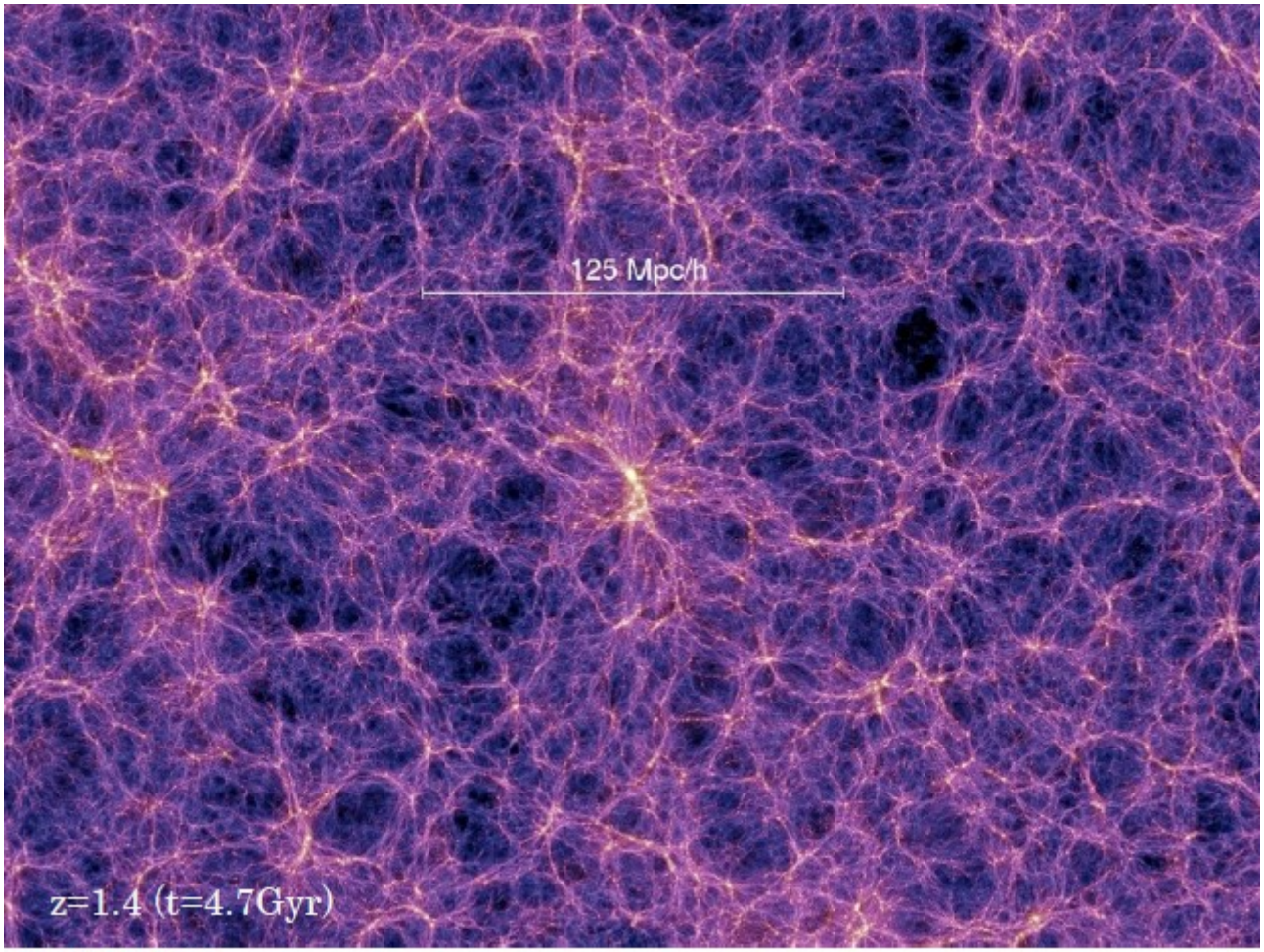

Galaxy map 3.8 billion years ago
Galaxy map 5.5 billion years ago
CMB 13.7 billion years ago

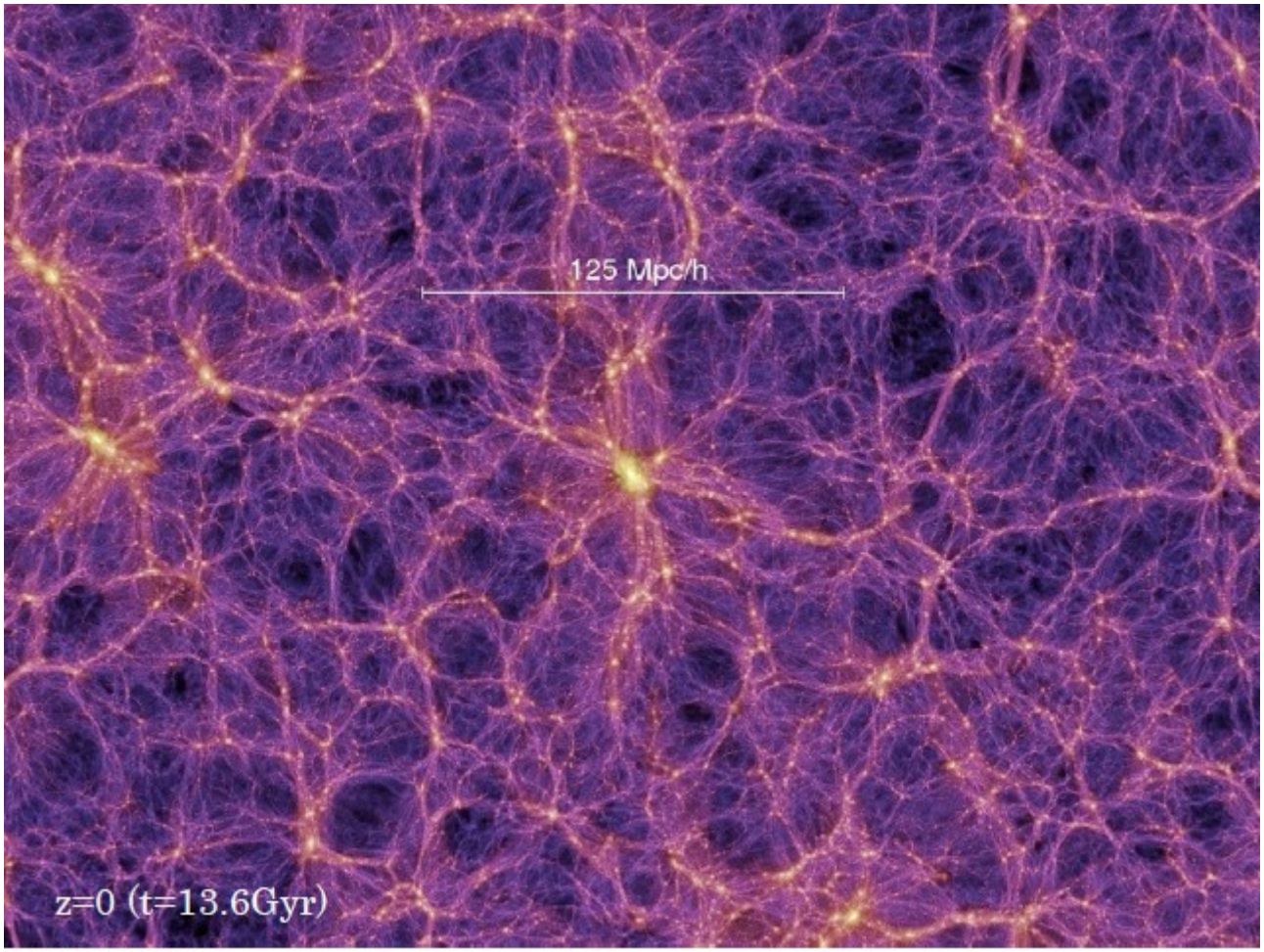
Mapping the expansion history:

Cosmology from the same physics imprinted in the galaxy distribution at different redshifts *if* BAO feature at late times same as initial


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)


125 Mpc/h
z=5.7 (t=1.0Gyr)


125 Mpc/h
z=1.4 (t=4.7Gyr)



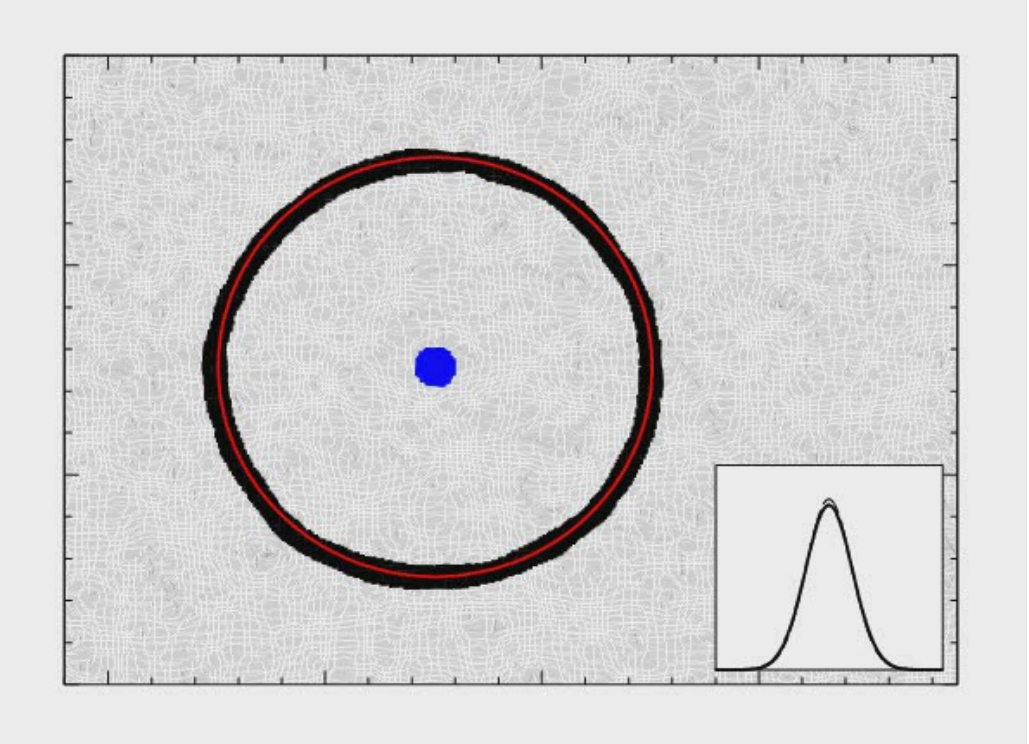
Tuesday, July 17, 2012

Springel et al. 2005

Although length ‘not’ affected, BAO ‘peak’ is smeared out (Bharadwaj 1996)

$x = q + S(t|q)$

S is shift from initial to final position. It is speed x time ~ Gaussian random number with rms ~7 Mpc



Padmanabhan et al. 2012

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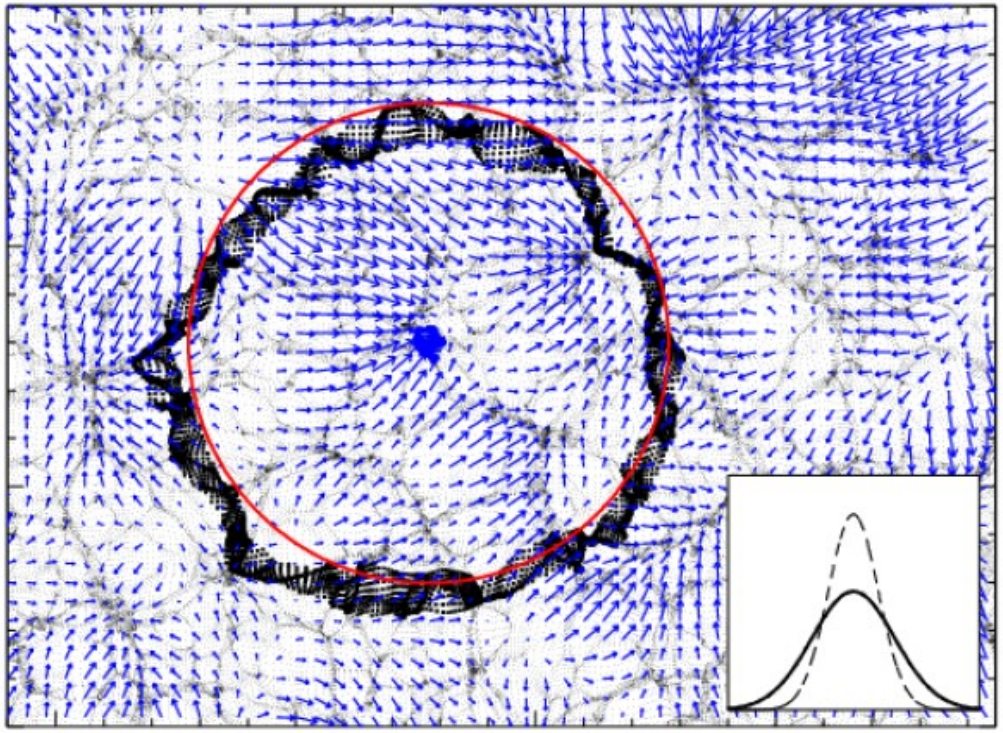
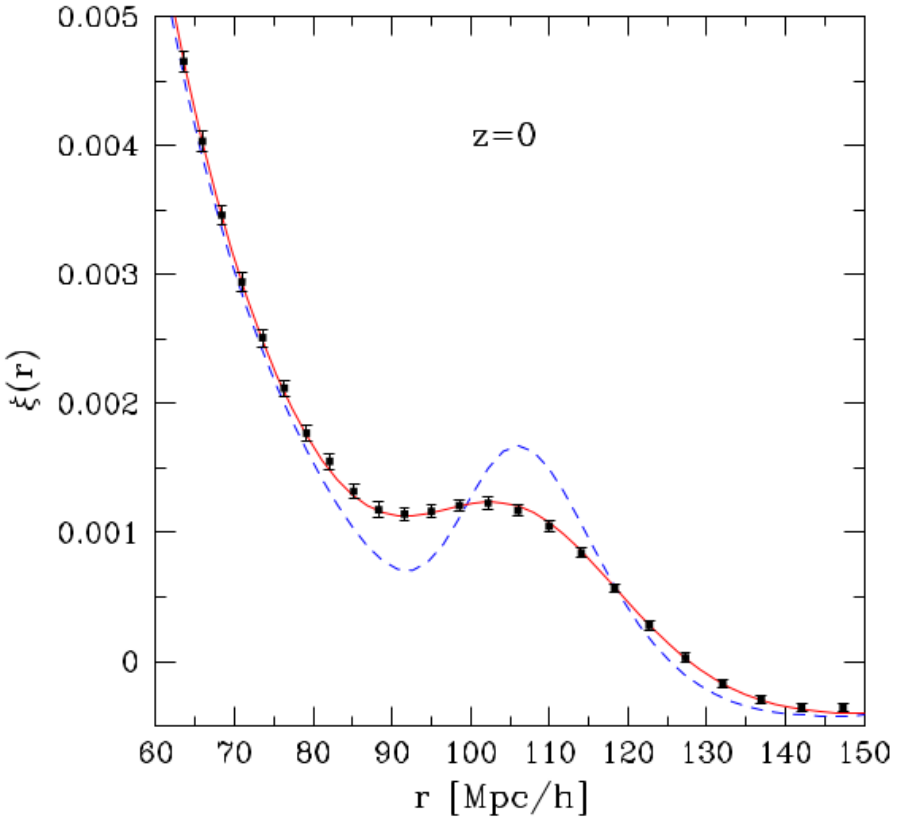
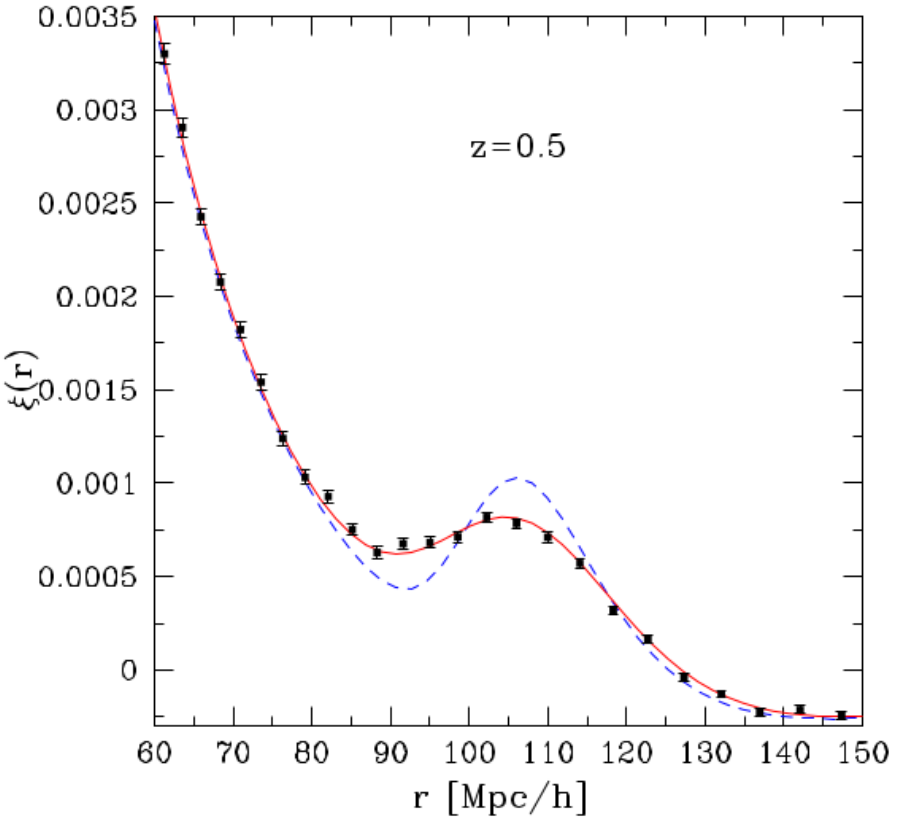


Padmanabhan et al. 2012

Smearing: $P(k) \sim P_{init}(k)\, e^{-k2\ smear2}$

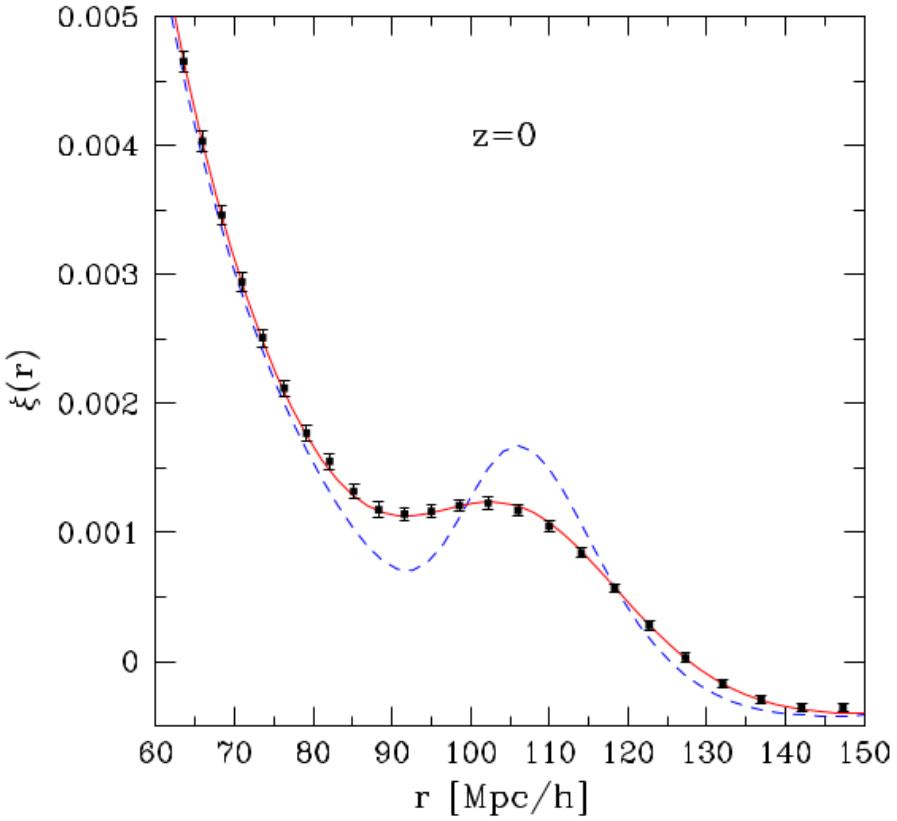
changes height alot, but scale ~ stable

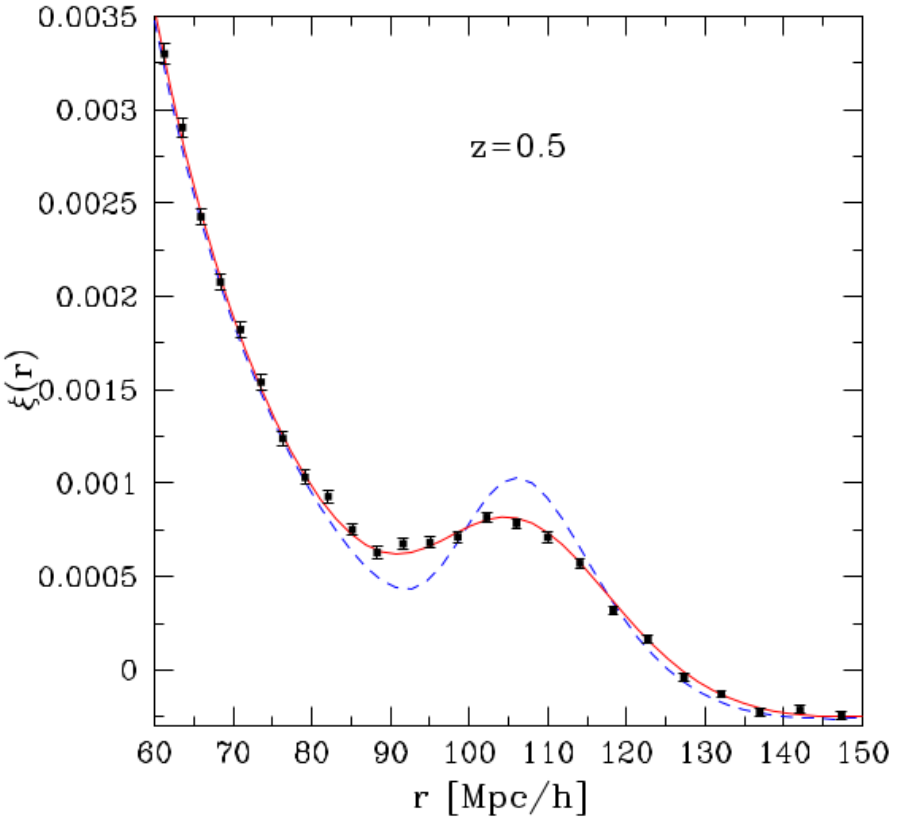




Crocce & Scoccimarro 2008

Can we use OT to undo smearing and ‘reconstruct’ the BAO feature?





Optimal Transport:

Assume initial density field uniform (same for all cosmologies); solve for displacements

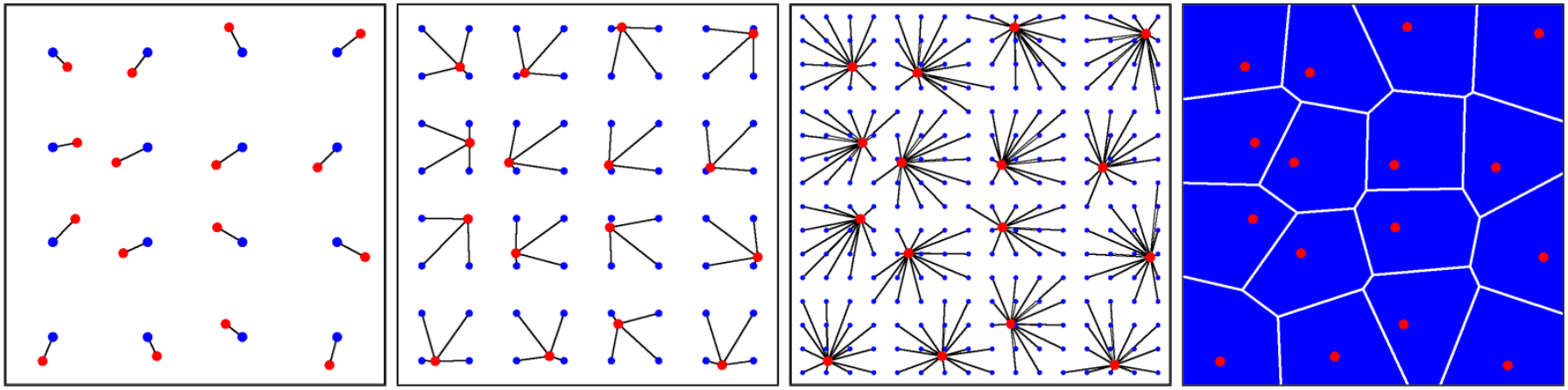


Figure 52: From discrete-discrete to semi-discrete optimal transport problem. The red points show the distribution of matter at current time and the blue points represent the initial condition by a regular grid. From left to right, we increase the precision by using a finer grid for the initial positions (Lévy et al., 2020).

Voronoi: same positions, volumes different

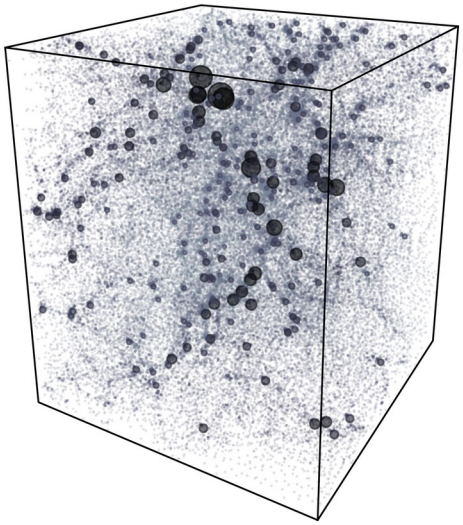
Laguerre: same volumes, positions different (= displacements)

halos (volume~mass) require 'weighted' semi-discrete OT

Optimal transport

(Nikakhtar et al. 2022, PRL)

$z = 0$

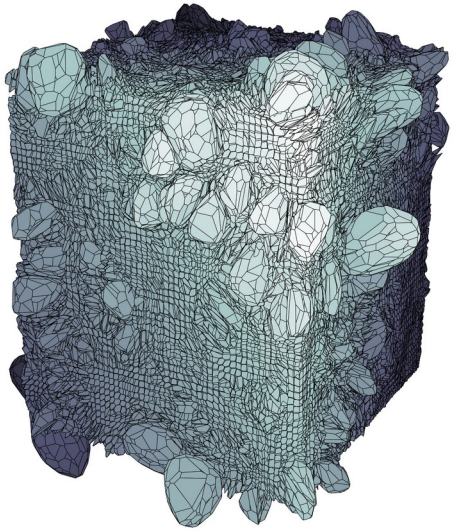


Weighted Semi-discrete OT Reconstruction:

Computing Laguerre cells V_i^{ψ}

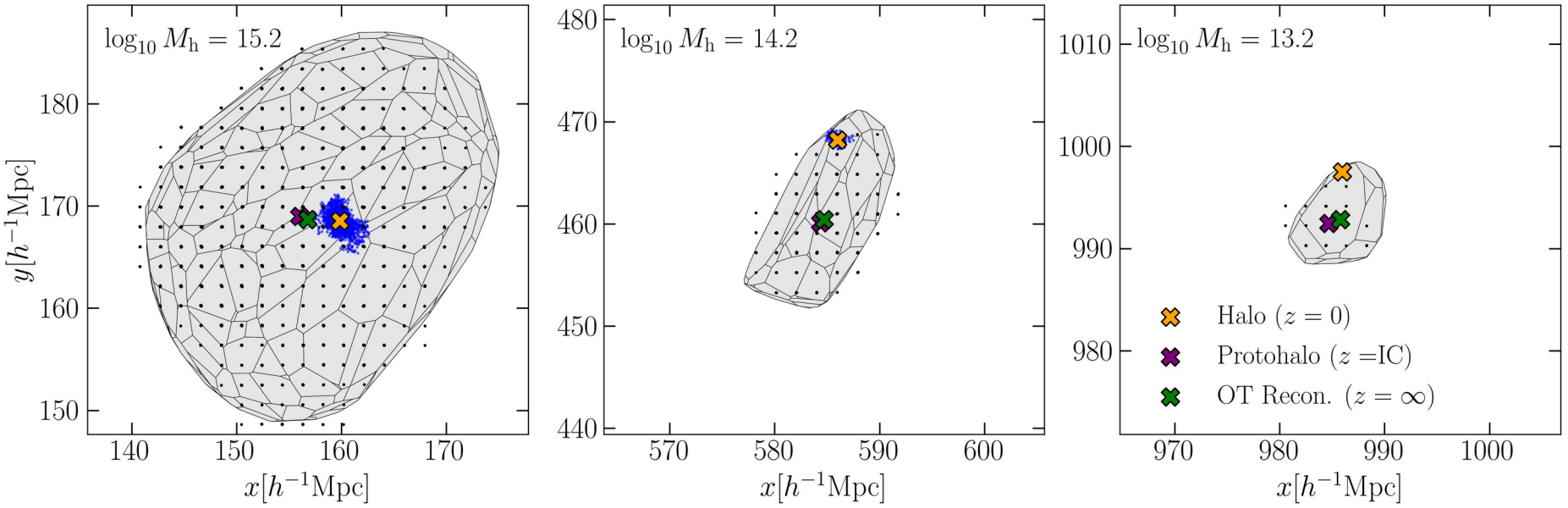
$$V_i^{\psi} = \left\{ \mathbf{q} \;\middle|\; \tfrac{1}{2}|\mathbf{x}_i - \mathbf{q}|^2 - \psi_i < \tfrac{1}{2}|\mathbf{x}_j - \mathbf{q}|^2 - \psi_j,\ \forall j \neq i \right\}$$

$z = \infty$



Reconstructs displacements,
hence protohalo positions, shapes

Optimal transport



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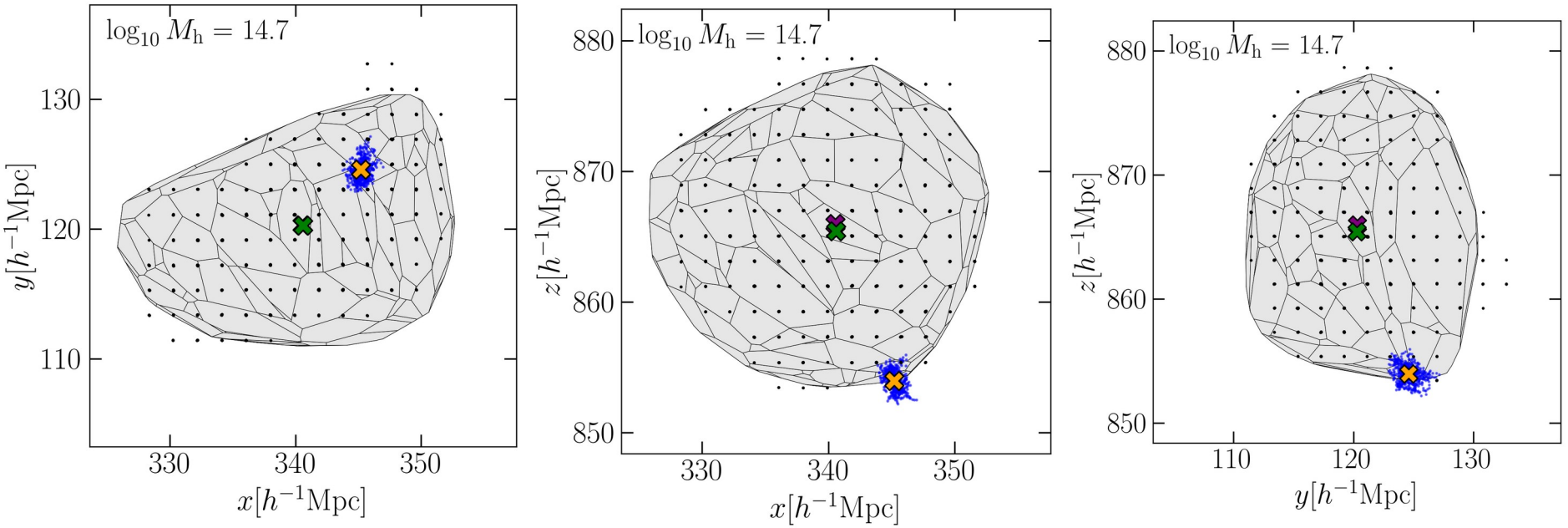
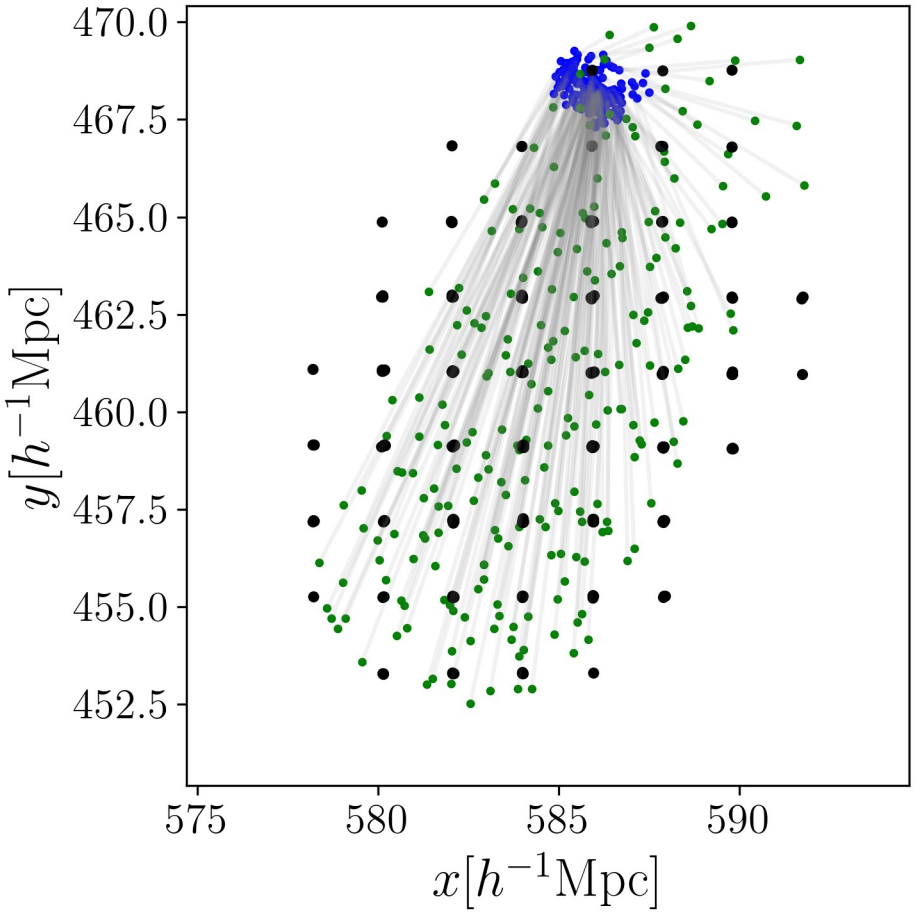


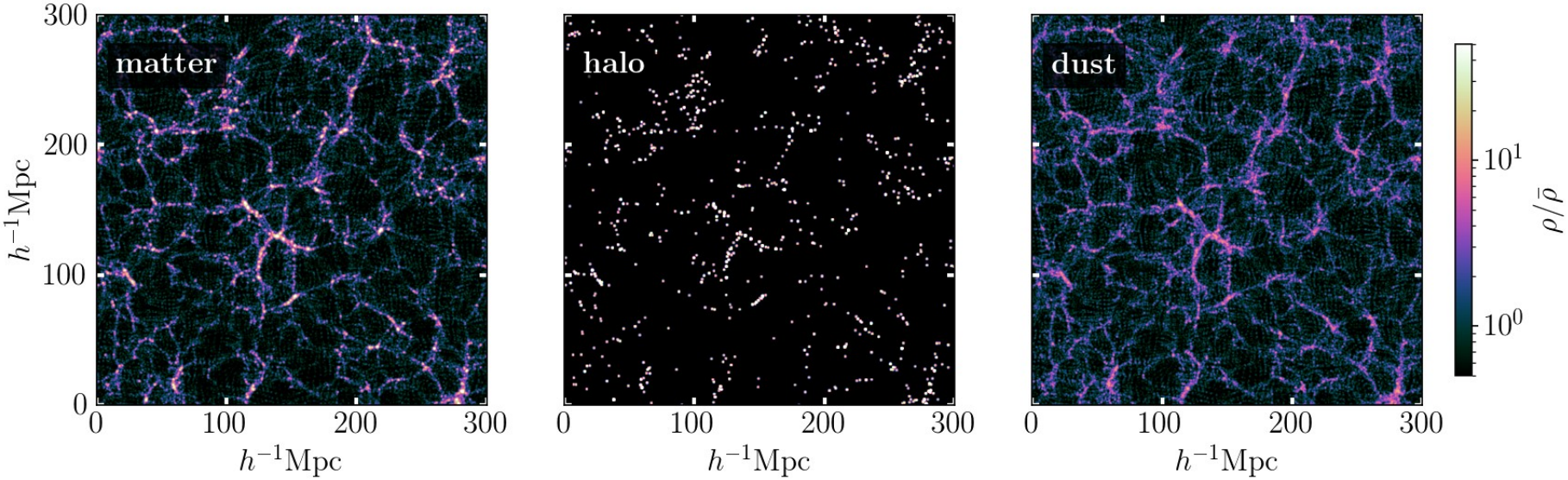
Reconstructs displacements,
hence protohalo positions, shapes

Optimal transport: self-consistency of weighting scheme

Shape reconstructed using wtSD-OT (~ protohalo grid) agrees with 'full' SD-OT



Optimal transport:
What if 'missing' some data



Model 'dust' using something simple?

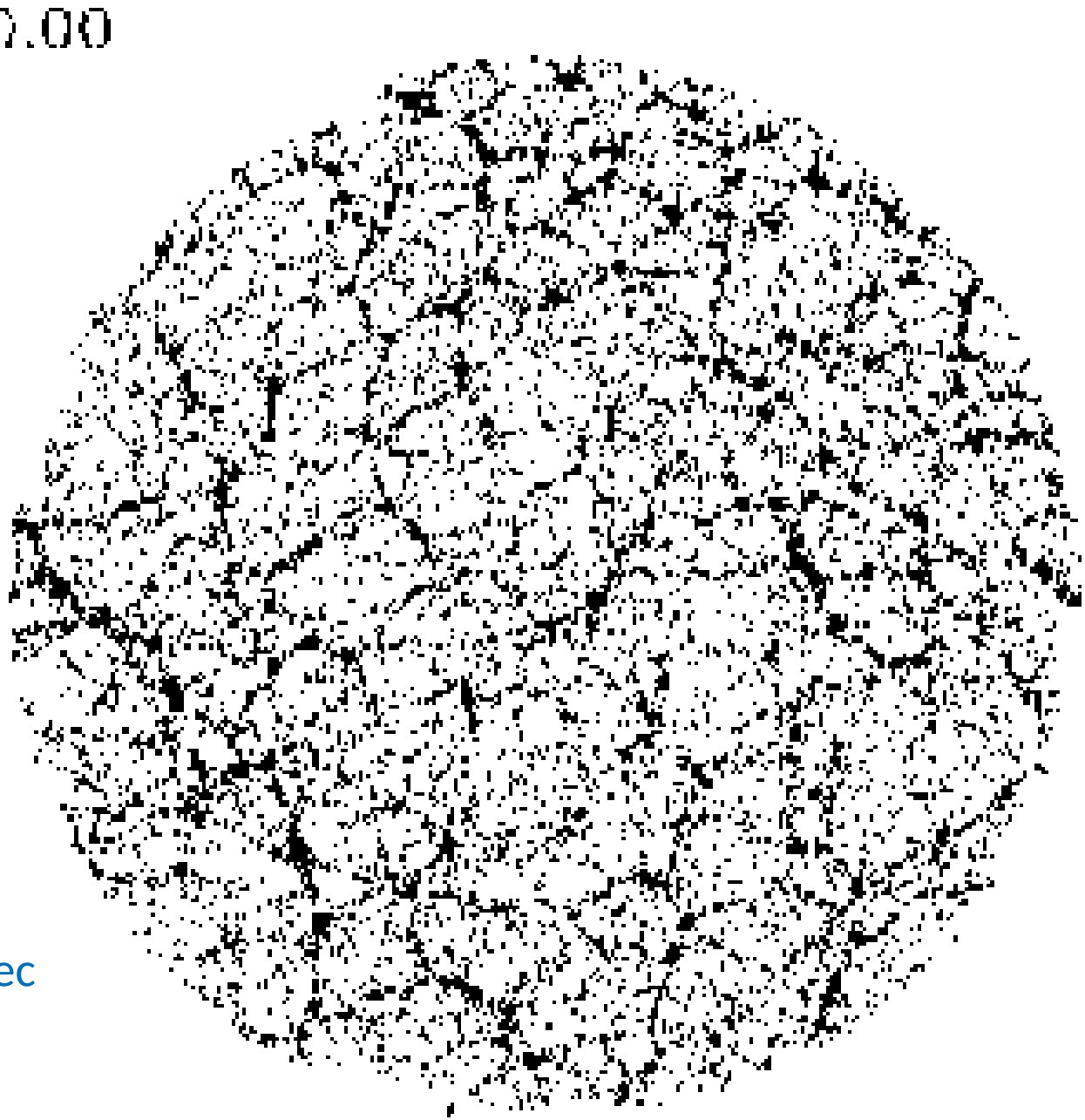
Use information about skeleton of cosmic web (ala Feldbrugge)?

Cosmology dependence?

Redshift space distortions:

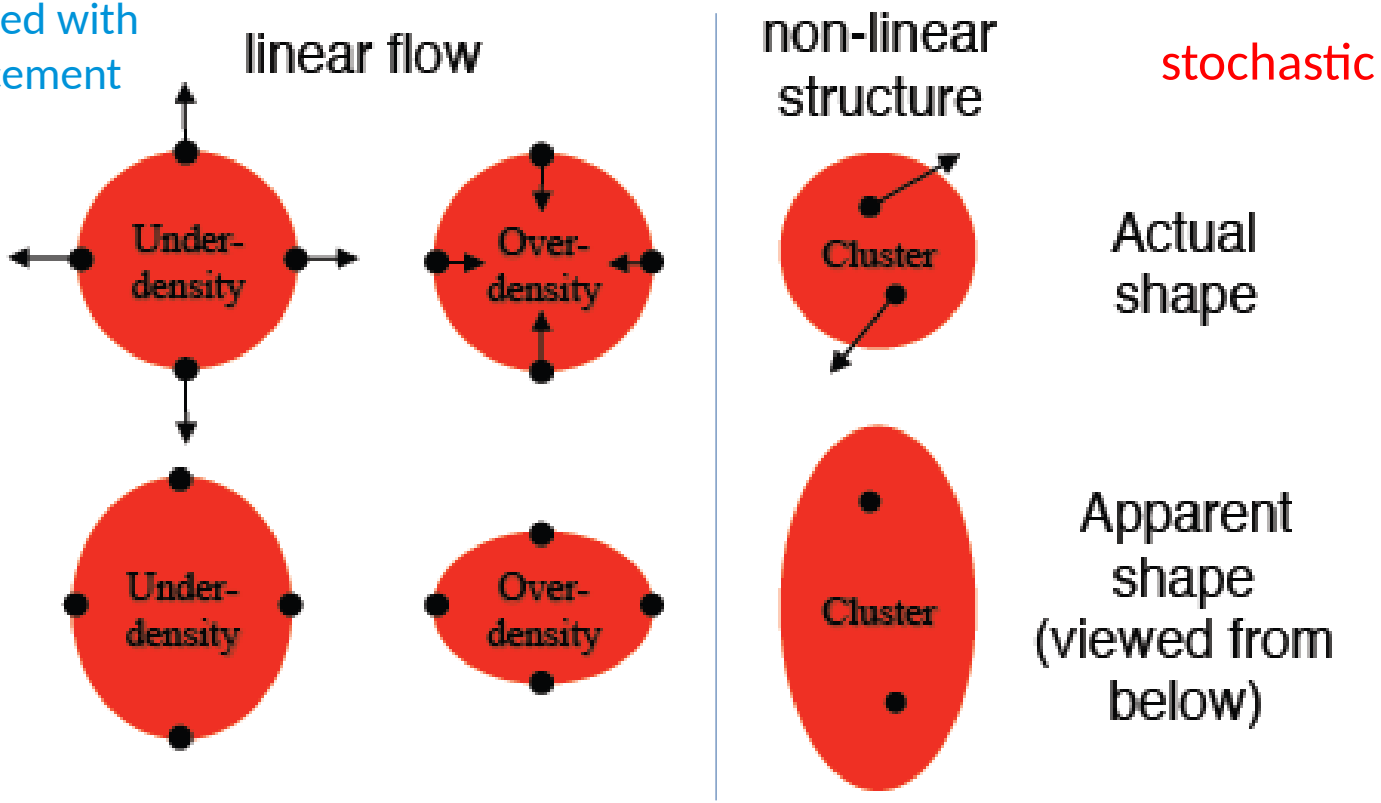
peculiar velocities driven by gravity

$cz_{obs} = Hd + v_{pec}$

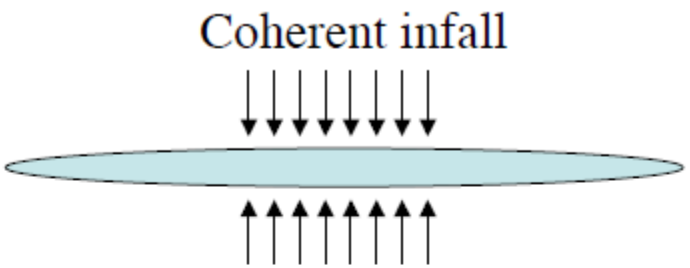


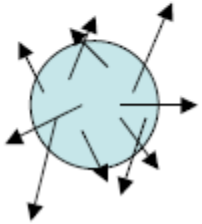
Two redshift space distortions: Linear + nonlinear

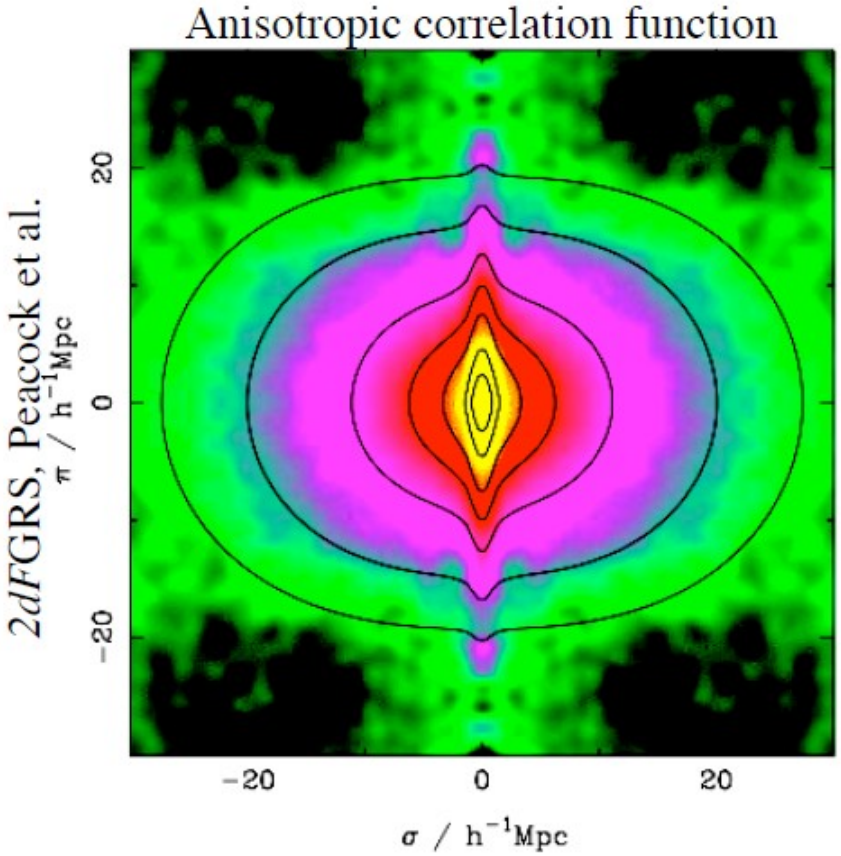
correlated with displacement



Redshift space distortions







$$1 + \xi_s(s_\parallel, s_\perp) = \int_{-\infty}^{\infty} dr_\parallel \, [1 + \xi(r)] \, \mathcal{P}(\underbrace{r_\parallel - s_\parallel}_{\mathbf{v}_p}, \mathbf{r})$$

On large scales, use Gaussian statistics to compute (Fisher 1995)

Linear redshift space distortions

- The displacements imply velocities which make redshift space position different from real space position

- $x_s = x + [v(x).d_{los}/|d_{los}|]/H$

  $= q + S(q) + [f\, S(q) \bullet d_{los}/|d_{los}|]$

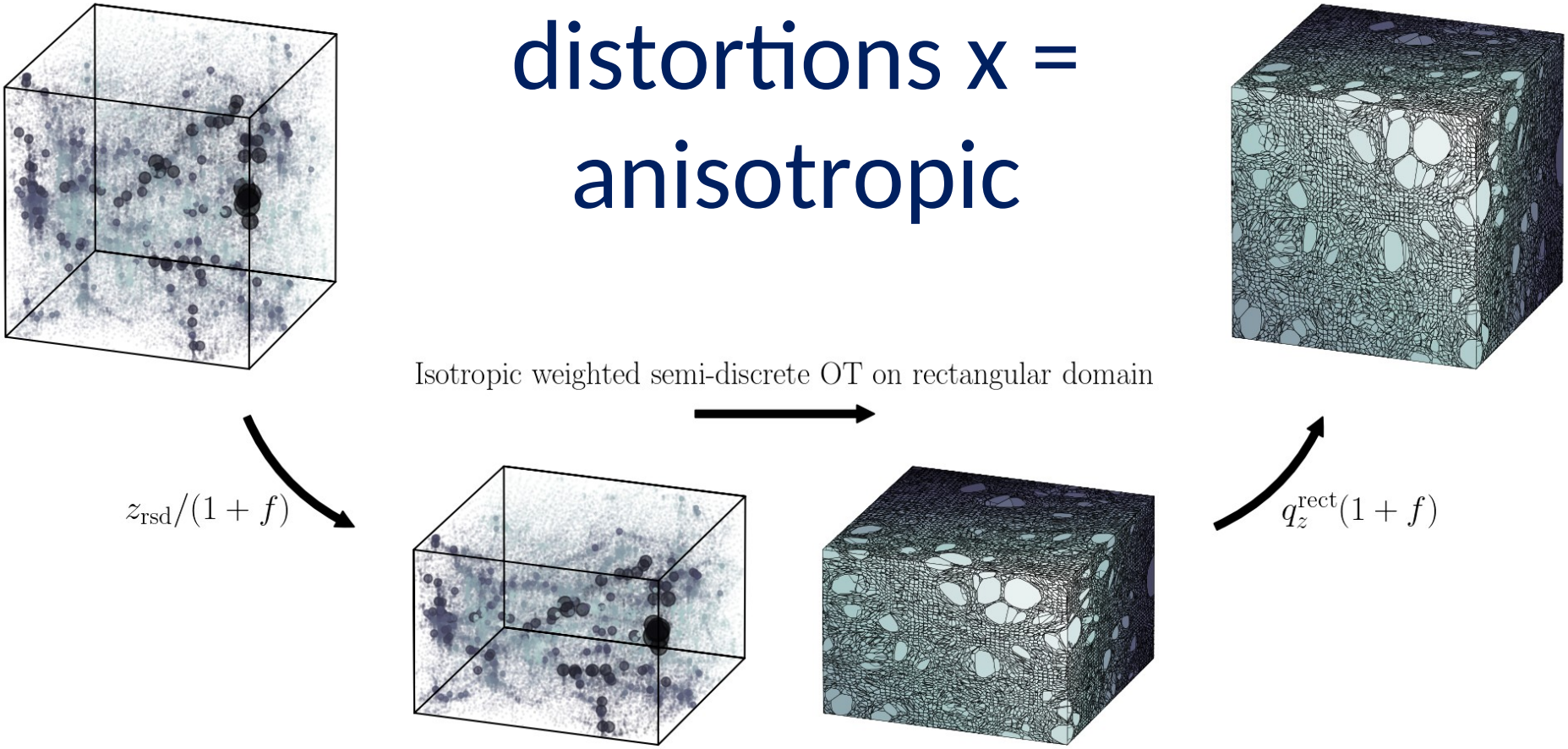
  If distances large, then line-of-sight is along z-coordinate

$$(x,y,z) = (q_x,q_y,q_z) + (S_x,S_y,S_z) + f\,(0,0,S_z)$$

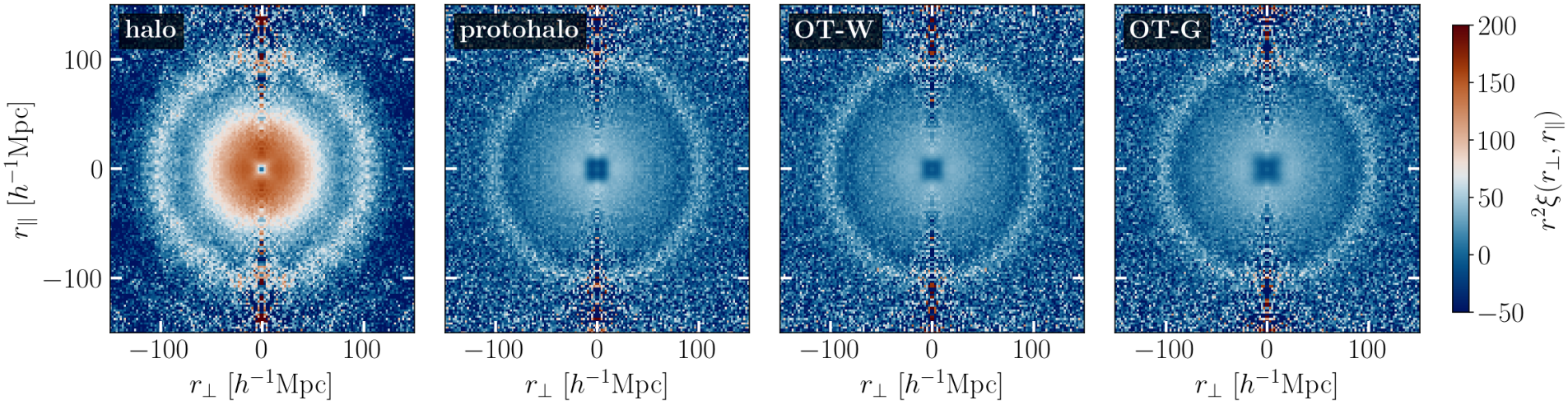
$$= (q_x,q_y,q_z) + [S_x,S_y,S_z(1+f)]$$

$$[S_x,S_y,S_z(1+f)] = (x-q_x,\ y-q_y,\ z-q_z)$$

Optimal transport: Redshift space distortions x = anisotropic

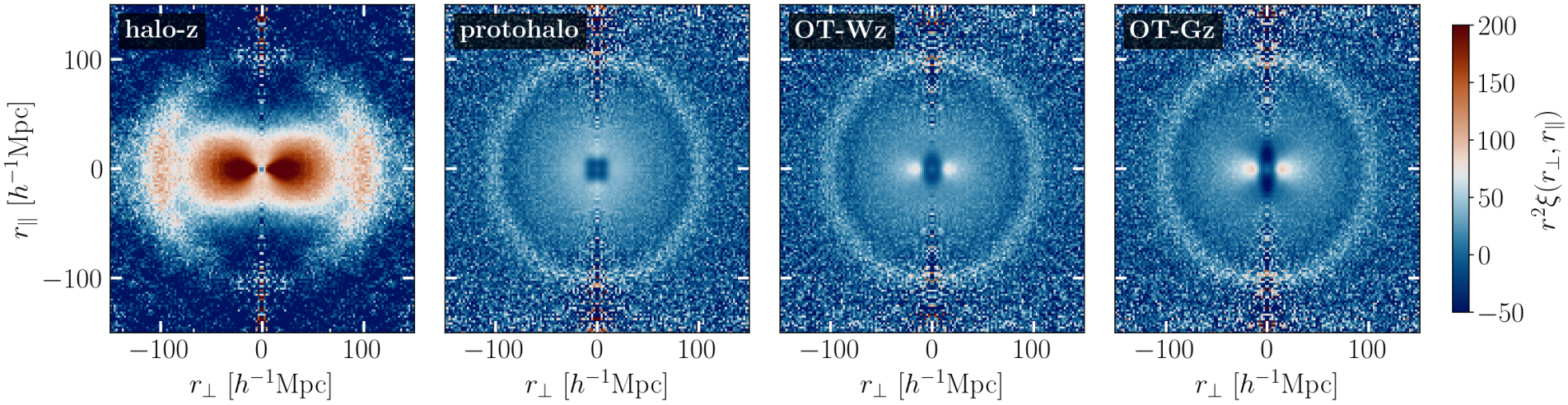


Optimal transport



Reconstructed 2-point statistics
similar to true initial statistics

Optimal transport



Reconstructed 2-point statistics
from RSD data much more isotropic
(e.g. reduces quadrupole)

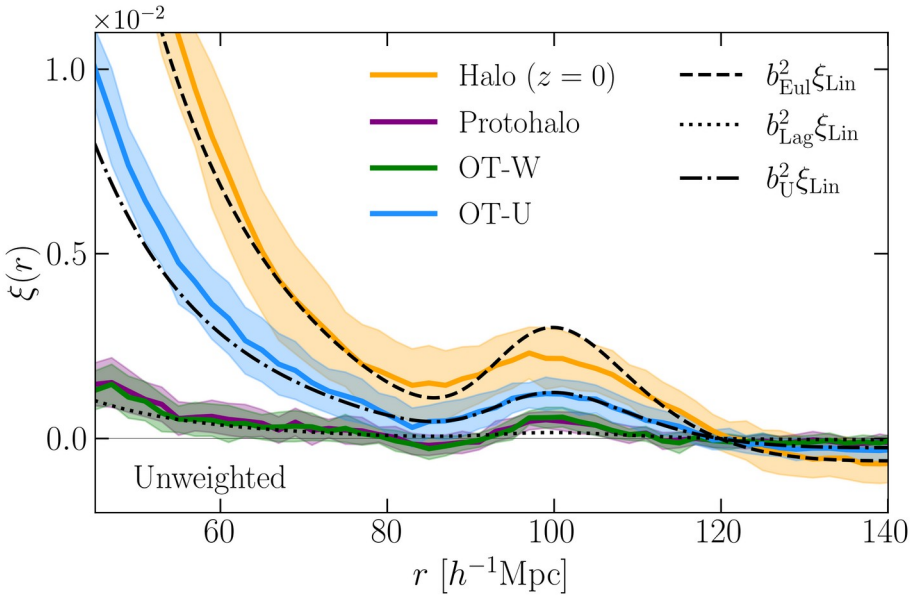
Motivation (Cosmology)

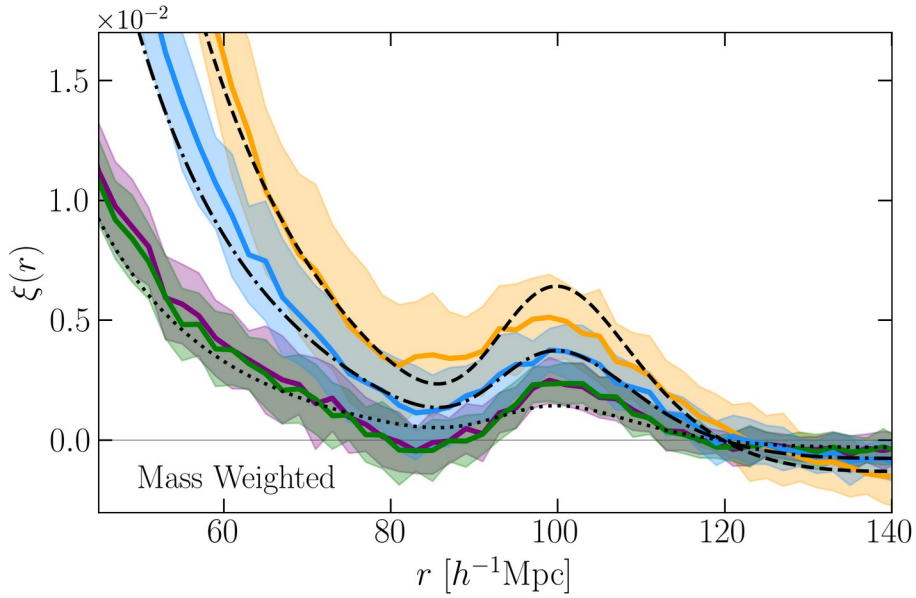
Reconstruction (OT)

Complications (Real-world)

- dust (missing data)
- mass weights (robust to errors)
- nonlinear RSD (noisy positions)
- filaments (= OT+adhesion?)

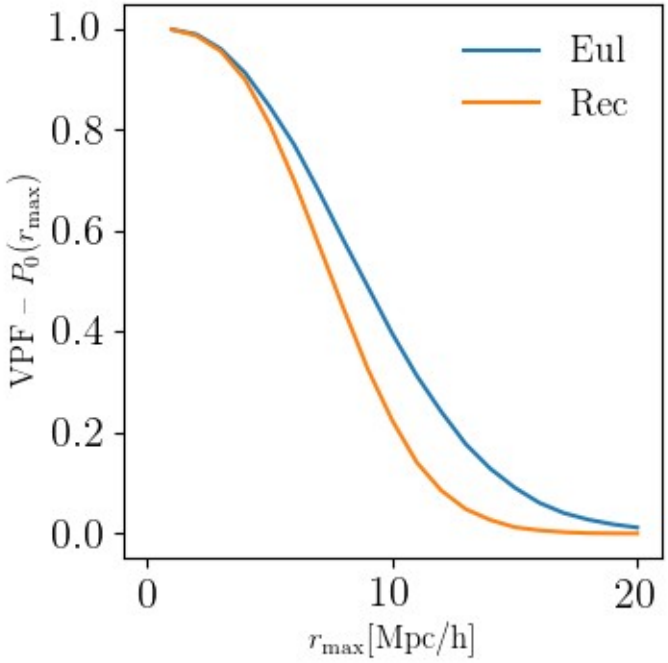
Optimal transport

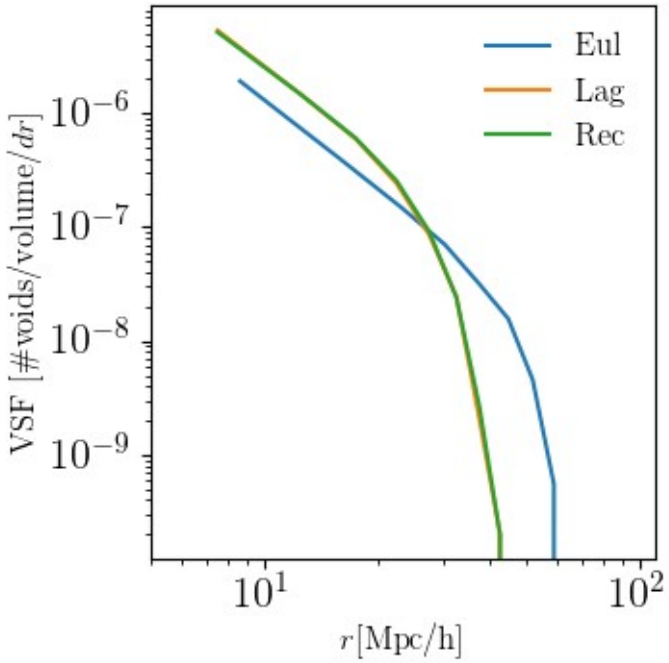




Reconstructs two-point statistics (mass weighted ~ HOD galaxies)

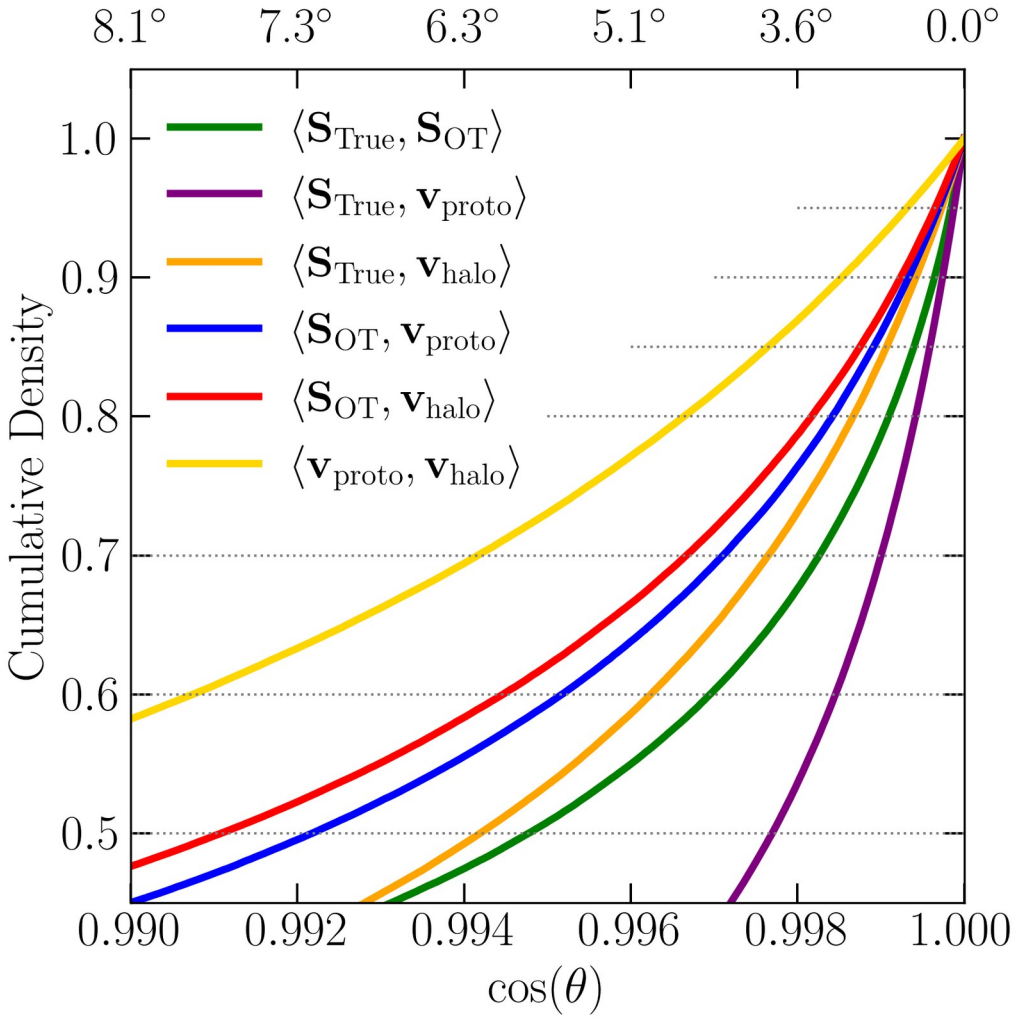
Optimal transport





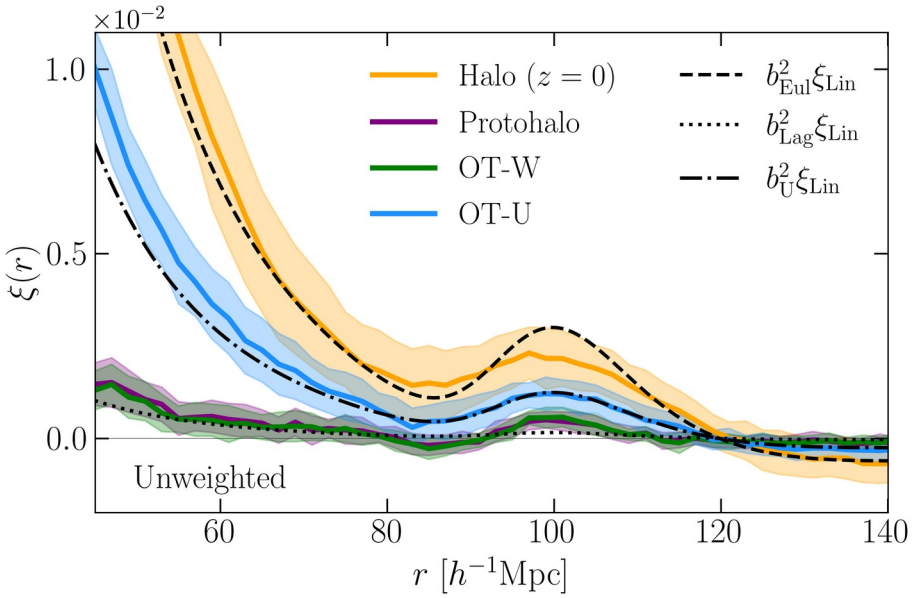
Reconstructs n-point statistics:
Void PDF (hence kNN), Void sizes

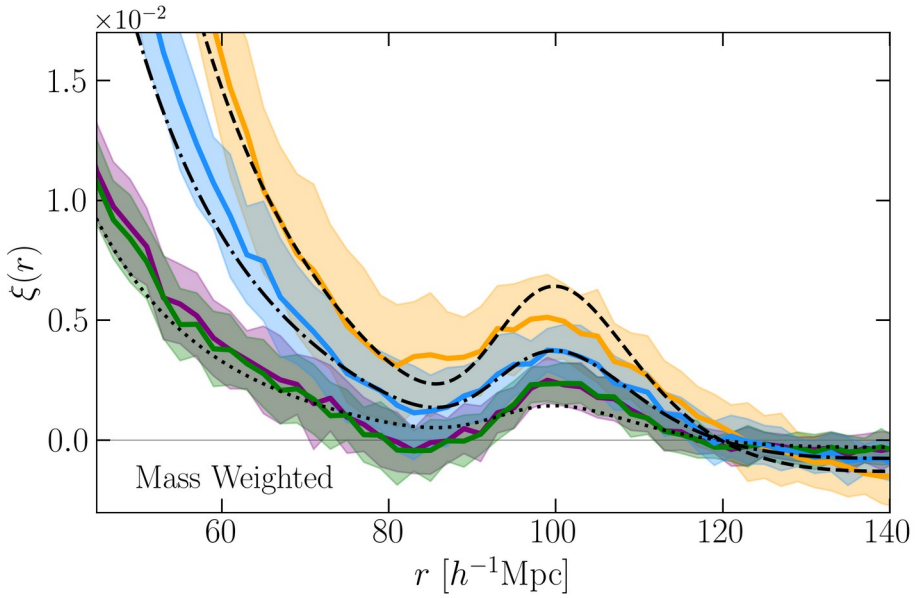
Optimal transport



Reconstructs displacements; BAO-kSZ synergy?

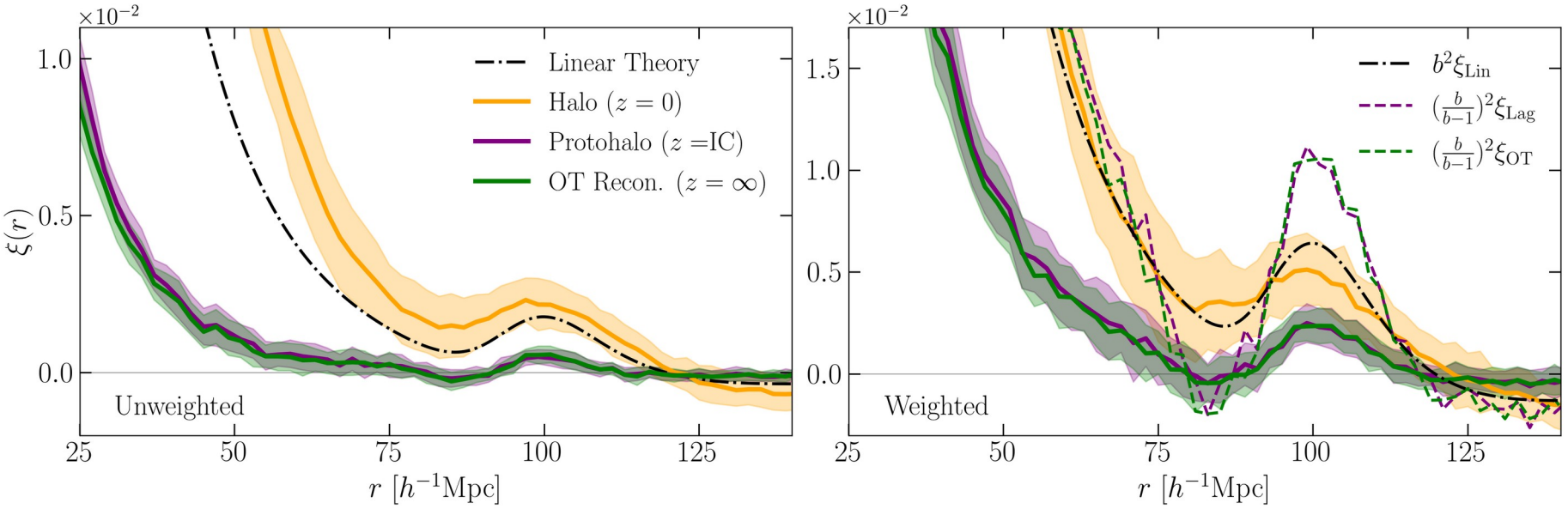
Optimal transport





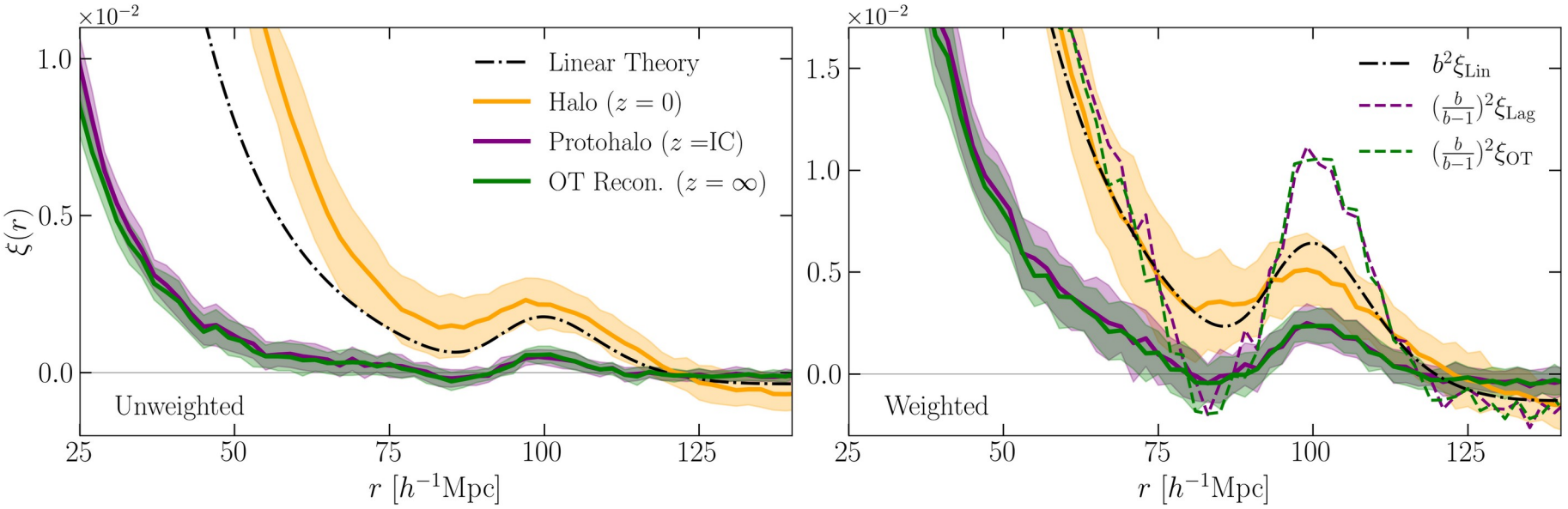
Reconstructs 2-point statistics (mass weighted ~ HOD)

Optimal transport



Reconstructed 2-point statistics scale as expected – enables determination of bias factor b

Optimal transport



Reconstructed 2-point statistics scale as expected – but shape different from pure linear theory!

Optimal transport